Design and Implementation of Speech Recognition Systems

Fall 2014
Ming Li

Class 5: Dynamic Time Warping-Recognizing speech
Sep 23 2014

Thanks to Professor Bhiksha Raj for the contribution of the slides
Speech Recognition by Template Matching

• Store “templates” for all words to be recognized
  – Template = example recording
    • Actually feature sequence from example recording

• Compute distance of input test data to all templates, select the closest

• Like spellchecking
Isolated word Speech Recognition

- Isolated word recognition scenario

Recordings (templates)

- Spoken input word
  - Word1
  - Word2
  - Word3
  - Word-N

compare

Best

• Isolated word recognition scenario
Speech Recognition as Template Matching

- Problem: Input and template may be different lengths

- Worse – the change in length may not be uniform

- Must nevertheless be able to say that the distance between the two above examples is small
  – Like string matching
DTW: DP for Speech Template Matching

- Back to template matching for text: *dynamic time warping*
  - Input and templates are sequences of feature vectors instead of letters

- Intuitive understanding of why DP-like algorithm might work to find a best alignment of a template to the input:
  - We need to search for a path that finds the following alignment:

```
<table>
<thead>
<tr>
<th>template</th>
<th>s</th>
<th>o</th>
<th>me</th>
<th>th</th>
<th>i</th>
<th>ng</th>
</tr>
</thead>
<tbody>
<tr>
<td>input</td>
<td>s</td>
<td>o</td>
<td>me</td>
<td>th</td>
<td>i</td>
<td>ng</td>
</tr>
</tbody>
</table>
```
  - The DP algorithm for text permits such alignments

- Consider the 2-D matrix of template-input frames of speech
Need to find something like this warped path
DTW: Adapting Concepts from DP

• Some concepts from string matching need to be adapted to this problem
  – What are the allowed set of transitions in the search trellis?
  – What are the edge and local node costs?
    • Nodes can also have costs

• Once these questions are answered, we can apply essentially the same DP algorithm to find a minimum cost match (path) through the search trellis
DTW: Adapting Concepts from DP

• What transitions are allowed..

• What is a “score”? 
DTW: Determining Transitions

- Transitions must account for stretching and shrinking of speech segments
  - To account for varying speech rates

- Unscored “Insertions” disallowed
  - Every input frame must be matched to some template frame
  - Different from Levenshtein distance computation where symbols were compared only at diagonal transitions

- For meaningful comparison of two different path costs, their lengths must be kept the same
  - So, every input frame is to be aligned to a template frame exactly once
  - Vertical transitions (mostly) disallowed
DTW: Transitions

- Typical transitions used in DTW for speech:

  1. The next input frame aligns to the same template frame as the previous one. (Allows a template segment to be arbitrarily stretched to match some input segment)

  2. The next input frame aligns to the next template frame. No stretching or shrinking occurs in this region

  3. The next input frame skips the next template frame and aligns to the one after that. Allows a template segment to be shrunk (by at most ½) to match some input segment

- Note that all transitions move one step to the right, ensuring that each input frame gets used exactly once along any path
Levenshtein vs. DTW: Transitions

• LEVENSHTEIN
  – Horizontal transition, no symbol comparison
  – Diagonal transition: Symbols are compared
  – Vertical transition: no symbol comparison

• DTW
  – Horizontal: symbol must be compared
  – Diagonal: Two varieties
    • Both require symbol comparison
  – Vertical: Disallowed
DTW: Use of Transition Types

- Short template, long input
- Approx. equal length template, input
- Long template, short input
• Other transition choices are possible:
  – Skipping more than one template frame (greater shrink rate)
  – Vertical transitions: the same input frame matches more than one template frame
    • This is less often used, as it can lead to different path lengths, making their costs not easily comparable
DTW: Local Edge and Node Costs

• Typically, there are no edge costs; any edge can be taken with no cost
• Local node costs measure the dissimilarity or distance between the respective input and template frames
• Since the frame content is a multi-dimensional feature-vector, what dissimilarity measure can we use?
• A simple measure is Euclidean distance; i.e. geometrically how far one point is from the other in the multi-dimensional vector space
  – For two vectors $X = (x_1, x_2, x_3 \ldots x_N)$, and $Y = (y_1, y_2, y_3 \ldots y_N)$, the Euclidean distance between them is:

$$\sqrt{\sum (x_i - y_i)^2}, \ i = 1 \ldots N$$

  – Thus, if $X$ and $Y$ are the same point, the Euclidean distance = 0
  – The farther apart $X$ and $Y$ are, the greater the distance
• Other distance measures could also be used:
  – Manhattan metric or the L1 norm: \( \Sigma |A_i - B_i| \)
  – Weighted Minkowski norms: \( (\Sigma w_i |A_i - B_i|^n)^{1/n} \)
DTW: Overall algorithm

- The transition structure and local edge and node costs are now defined
- The search trellis can be realized and the DP algorithm applied to search for the minimum cost path, as before
  - Example trellis using the transition types shown earlier:
DTW: Overall algorithm

• The best path score can be computed using DP as before
  – But the best path score must now consider both node and edge scores
  – Each node is a comparison of a vector from the data against a vector from the template
DTW: Overall Algorithm

- $P_{i,j}$ = best path cost from origin to node $[i,j]$
  - $i$-th template frame aligns with $j$-th input frame
- $C_{i,j}$ = local node cost of aligning template frame $i$ to input frame $j$

\[
P_{i,j} = \min (P_{i,j-1} + C_{i,j}, \ P_{i-1,j-1} + C_{i,j}, \ P_{i-2,j-1} + C_{i,j})
\]

\[
= \min (P_{i,j-1}, \ P_{i-1,j-1}, \ P_{i-2,j-1}) + C_{i,j}
\]

- Edge costs are 0 in above formulation
DTW: Overall Algorithm

- If the template is $m$ frames long and the input is $n$ frames long, the best alignment of the two has the cost = $P_{m,n}$

- The computational is proportional to:
  $M \times N \times 3$, where
  $M =$ No. of frames in the template
  $N =$ No. of frames in the input
  3 is the number of incoming edges per node
Handling Surrounding Silence

- The DTW algorithm automatically handles any silence region surrounding the actual speech, within limits:

- But, the transition structure does not allow a region of the template to be shrunk by more than ½!
  - Need to ensure silences included in recording are of generally consistent lengths, or allow other transitions to handle a greater “warp”
Isolated Word Recognition Using DTW

• We now have all ingredients to perform isolated word recognition of speech

• “TRAINING”: For each word in the vocabulary, pre-record a spoken example (its template)

• RECOGNITION of a given recording:
  – For each word in the vocabulary
    • Measure distance of recording to template using DTW
  – Select word whose template has smallest distance
Recognition

- For each template:
  - Create a trellis against data
    - Figure above assumes 7 vectors in the data
  - Compute the cost of the best path through the trellis

- Select word corresponding to template with lowest best path cost
**Time Synchronous Search**

- Match all templates Synchronously
- STACK trellises for templates above one another
  - Every template match is started simultaneously and stepped through the input in lock-step fashion
    - Hence the term *time synchronous*

- Advantages
  - No need to store the entire input for matching with successive templates
  - Enables realtime: Matching can proceed as the input arrives
  - Enables *pruning* for computational efficiency
Example: Isolated Speech Based Dictation

- We could, in principle, almost build a large vocabulary isolated-word dictation application using the techniques learned so far

- Training: Record templates (i.e. record one or more instance) of each word in the vocabulary

- Recognition
  - Each word is spoken in isolation, \textit{i.e.} silence after every word
  - Each isolated word compared to all templates
    - Accuracy would probably be terrible

- Problem: How to detect when a word is spoken?
  - Explicit “click-to-speak”, “click-to-stop” button clicks from user, for every word?
    - Obviously extremely tedious
  - Need a speech/silence detector!
Endpointing: A Revision

- Goal: automatically detect pauses between words
  - to segment the speech stream into isolated words?

- Such a speech/silence detector is called an endpointer
  - Detects speech/silence boundaries (shown by dotted lines)

- Most speech applications use such an endpointer to relieve the user of having to indicate start and end of speech
A Simple Endpointing Scheme

- Based on silence segments having low signal amplitude
  - Usually called energy-based endpointing

- Audio is processed as a short sequence of frames
  - Exactly as in feature extraction

- The signal energy in each frame is computed
  - Typically in decibels (dB): $10 \log (\Sigma x_i^2)$, where $x_i$ are the sample values in the frame

- A threshold is used to classify each frame as speech or silence
- The labels are smoothed to eliminate spurious labels due to noise
  - E.g. minimum silence and speech segment length limits may be imposed
  - A very short speech segment buried inside silence may be treated as silence

- The above should now make sense to you if you’ve completed the feature computation code
Speech-Silence Detection: Endpoint

- The computed “energy track” shows signal power as a function of time
- A simple threshold can show audio segments
  - Can make many errors though
- What is the optimal threshold?
Speech-Silence Detection: Endpointer

- Optimal threshold: Find average value of latest contiguous non-speech segment of minimum length
- Find average energy value in the segment
  - \( \text{Avgnoiseegy} = \frac{1}{\text{Ncontiguous frames}} \times \sum(\text{energy of frames}) \)
- Average noise energy plus threshold = speech threshold
  - \( E_{\text{gy}} > \alpha \times \text{Avgnoiseegy} \)
  - \( \alpha \) typically > 6dB
Speech-Silence Detection: Endpoint

- Alternative strategy: TWO thresholds
  - Onset of speech shows sudden increase in energy

- Onset threshold: $\text{avgnoiseeeg} \times \alpha$
  - Speech detected if frame energy $> \text{onset threshold}$
  - $\alpha > 12\text{dB}$

- Offset threshold: $\text{avgnoiseeeg} \times \beta$
  - $\beta > 6\text{dB}$

- Speech detected between onset and offset
  - Additional smoothing of labels is still required
  - Typically, detected speech boundaries are shifted to include 200ms of silence either side
Isolated Speech Based Dictation (Again)

• With such an endpointer, we have all the tools to build a complete, isolated word recognition based dictation system, or any other application

• However, as mentioned earlier, accuracy is a primary issue when going beyond simple, small vocabulary situations
Dealing with Recognition Errors

- Applications can use several approaches to deal with speech recognition errors

- Primary method: improve performance by using better models in place of simple templates
  - We will consider this later

- However, most systems also provide other, orthogonal mechanisms for applications to deal with errors
  - Confidence estimation
  - Alternative hypotheses generation (N-best lists)

- We now consider these two mechanisms, briefly
Confidence Scoring

- **Observation**: DP or DTW will *always* deliver a minimum cost path, *even if it makes no sense*
- Consider string matching:

<table>
<thead>
<tr>
<th>templates</th>
<th>min. edit distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yesterday</td>
<td></td>
</tr>
<tr>
<td>Today</td>
<td></td>
</tr>
<tr>
<td>Tomorrow</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>input</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>5</td>
</tr>
<tr>
<td>Yesterday</td>
<td>7</td>
</tr>
</tbody>
</table>

- The template with minimum edit distance will be chosen, even though it is “obviously” incorrect
  - How can the application discover that it is “obviously” wrong?
- *Confidence scoring* is the problem of determining how confident one can be that the recognition is “correct”
Confidence Scoring for String Match

• A simple confidence scoring scheme: Accept the matched template string only if the cost $\leq$ some threshold
  – We encountered its use in the spell checking example!

• Accept if no. of errors is below some fixed threshold

• Or: Accept if cost $\leq 1 + \text{some fraction (e.g. 0.1)}$ of template string length
  – Templates of 1-9 characters tolerate 1 error
  – Templates of 10-19 characters tolerate 2 errors, etc.

• Easy to think of other possibilities, depending on the application

• Confidence scoring is one of the more application-dependent functions in speech recognition
Confidence Scoring for DTW

• Similar thresholding technique for template matching by DTW?
  – Unlike in string matching, the cost measures are not immediately, meaningfully “accessible” values
  – Need to know range of minimum cost when correctly matched and when incorrectly matched
    • If the ranges do not overlap, one could pick a threshold

![Diagram showing distribution of DTW costs for correctly and incorrectly identified templates with an overlap region susceptible to classification errors.](image-url)
Confidence: Procedure

- “Recognize” many many “development” recordings
  - Several will be recognized correctly
  - Others will be recognized wrongly

- Training confidence classifier
  - Distribution of scores of all wrongly recognized utterances
  - Distribution of scores of all correctly recognized utterances

- Confidence on test recording:
  - Option 1: Find optimal threshold for correct vs. wrong
  - Option 2: Compute confidence score = \( \frac{P(\text{test} \mid \text{correct})}{P(\text{test} \mid \text{error})} \)
Confidence Scoring for DTW

• As with string matching, DTW cost must be *normalized*
  – Use DTW cost / frame of input speech, instead of total DTW cost, before determining threshold

• Cost distributions and threshold have to be determined *empirically*, based on a sufficient collection of test data

• Unfortunately, confidence scores based on such distance measures are not very reliable
  – Too great an overlap between distribution of scores for correct and incorrect templates
  – We will see other, more reliable methods later on
N-best List Generation

- *Example*: Powerpoint catches spelling errors and offers several alternatives as possible corrections
- *Example*: In *Dragon Dictate*, one can select a recognized word and obtain alternatives
  - Useful if the original recognition was incorrect

- Basic idea: identifying not just the best match, but the top so many matches; *i.e.*, the *N-best list*

- Not hard to guess how this might be done, either for string matching or isolated word DTW!
  - (How?)
N-best List

- Match all templates
- RANK the words (templates) by the minimum-cost-path score for the template/trellis
- Return top-N words in order of minimum cost
Improving Accuracy: Multiple Templates

• Problems with using a single exemplar as a template
  – A single template will not capture all variations in the manner of saying a word
    • Works poorly even for a single speaker
    • Works very poorly across different speakers

• Use multiple templates for each word to handle the variations
  – Preferably collected from several speakers

• Template matching algorithm is easily modified
  – Simply match against all available templates and pick the best

• However, computational cost of matching increases linearly with the number of available templates
Reducing Search Cost: Pruning

• Reducing search cost implies reducing the size of the lattice that has to be evaluated

• As in string matching, there are several ways to accomplish this
  – Reducing the size of the models (templates)
    • *E.g.* replacing the multiple templates for a word by a single, *average* one
    • Reducing allowed transitions
  – Eliminating parts of the lattice from consideration altogether
    • *search pruning*, or just *pruning*
  – We consider search pruning first

• Basic consideration in pruning: *As long as the best cost path is not eliminated by pruning, we obtain the same result*
Pruning by Limiting Search Paths

- Assume that the input and the best matching template do not differ significantly from each other
  - For speech, equivalent to assuming the speaking rate is similar for the template and the input
  - The best path matching the two will lie close to the “diagonal”
- Thus, we need not search far off the diagonal. If the search-space “width” is kept constant, cost of search is linear in utterance length instead of quadratic
- However, errors occur if the speaking rate assumption is violated
  - i.e. if the template needs to be warped more than allowed by the width
Pruning by Limiting Search Paths

• What are problems with this approach?
Pruning by Limiting Search Paths

• What are problems with this approach?
  – Text: With lexical tree models, the notion of “diagonal” becomes difficult
  – For speech too there is no clear notion of a diagonal in most cases
    • As we shall see later
Pruning by Limiting Path Cost

- **Observation**: Partial paths that have “very high” costs will rarely recover to win.
- Hence, poor partial paths can be eliminated from the search:
  - For each frame $j$, after computing all the trellis nodes path costs, determine which nodes have too high costs.
  - Eliminate them from further exploration.
  - *(Assumption: In any frame, the best partial path has low cost)*
- **Q**: How do we define “high cost”?

![Diagram showing partial paths and high cost partial paths]

High cost partial paths (red);
Do not explore further

Partial best paths
Pruning by Limiting Path Cost

• As with confidence scoring, one could define high path cost as a value worse than some fixed threshold
  – But, as already noted, absolute costs are unreliable indicators of correctness
  – Moreover, path costs keep increasing monotonically as search proceeds
    • Recall the path cost equation
      \[ P_{i,j} = \min (P_{i,j-1}, P_{i-1,j-1}, P_{i-2,j-1}) + C_{i,j} \]

• Fixed threshold will not work
Pruning : Fixed Width Pruning

- Retain only the K best nodes in any column
  - K is the “fixed” beam width

With K = 2
The two best scoring nodes are retained
Fixed Width Pruning

• Advantages
  – Very predictable computation
    • Only K nodes expand out into the future at each time.

• Disadvantage
  – Will often prune out correct path when there are many similar scoring paths
  – In time-synchronous search, will often prune out correct template
Pruning: **Beam Search**

- In each frame \( j \), set the pruning threshold by a fixed amount \( T \) relative to the best cost in that frame
  - *I.e.* if the best partial path cost achieved in the frame is \( X \), prune away all nodes with partial path cost > \( X+T \)
  - Note that *time synchronous* search is very efficient for implementing the above

- Advantages:
  - Unreliability of absolute path costs is eliminated
  - Monotonic growth of path costs with time is also irrelevant

- Search that uses such pruning is called *beam search*
  - This is the most widely used search optimization strategy

- The relative threshold \( T \) is usually called “*relative beam width*” or just *beam width* or *beam*
Beam Search Visualization

- The set of lattice nodes actually evaluated is the *active* set.
- Here is a typical “map” of the *active region*, aka *beam* (confusingly).

![Active Region Diagram]

- Presumably, the best path lies somewhere in the active region.
Unlike the fixed width approach, the computation reduction with beam search is unpredictable.

- The set of active nodes at frames $j$ and $k$ is shown by the black lines.

However, since the active region can follow any warping, it is likely to be relatively more robust and efficient than the fixed width approach.
Determining the Optimal Beam Width

• Determining the optimal beam width to use is crucial
  – Using too narrow or tight a beam (too low $T$) can prune the best path and result in too high a match cost, and errors
  – Using too large a beam results in unnecessary computation in searching unlikely paths
  – One may also wish to set the beam to limit the computation (e.g. for real-time operation), regardless of recognition errors

• Unfortunately, there is no mathematical solution to determining an optimal beam width

• Common method: Try a wide range of beams on some test data until the desired operating point is found
  – Need to ensure that the test data are somehow representative of actual speech that will be encountered by the application
  – The operating point may be determined by some combination of recognition accuracy and computational efficiency
Determining the Optimal Beam Width

- Any value around the point marked $T$ is a reasonable beam for minimizing *word error rate* (WER)
- A similar analysis may be performed based on average CPU usage (instead of WER)
• Thus far, we considered beam search to prune search paths within a single template

• However, its strength really becomes clear in actual recognition (i.e. time synchronous search through all templates simultaneously)
  – In each frame, the beam pruning threshold is determined from the *globally* best node in that frame (from all templates)
  – Pruning is performed globally, based on this threshold
Beam Search Applied to Recognition

• Advantage of simultaneous time-synchronous matching of multiple templates:
  – Beams can be globally applied to all templates
  – We use the best score of all template frames (trellis nodes at that instant) to determine the beam at any instant
  – Several templates may in fact exit early from contention

• In the ideal case, the computational cost will be independent of the number of templates
  – All competing templates will exit early
  – Ideal cases don’t often occur
Pruning and Dynamic Trellis Allocation

• Since any form of pruning eliminates many trellis nodes from being expanded, there is no need to keep them in memory
  – Trellis nodes and associated data structures can be allocated *on demand* (*i.e.* whenever they become active)
  – This of course requires some book-keeping overhead

• May not make a big difference in small vocabulary systems
• But pruning is an essential part of all medium and large vocabulary systems
  – The search trellis structures in 20k word applications take up about 10MB with pruning
  – Without pruning, it could require more than 10 times as much!
Recognition Errors Due to Pruning

• Speech recognition invariably contains errors

• Major causes of errors:
  – Inadequate or inaccurate models
    • Templates may not be representative of all the variabilities in speech
  – Search errors
    • Even if the models are accurate, search may have failed because it found a sub-optimal path

• How can our DP/DTW algorithm find a sub-optimal path?
  – Because of pruning: it eliminates paths from consideration based on local information (the pruning threshold)

• Let $W$ be the best cost word for some utterance, and $W'$ the recognized word (with pruning)
  – In a full search, the path cost for $W$ is better than for $W'$
  – But if $W$ is not recognized when pruning is enabled, then we have a pruning error or search error
Measuring Search Errors

• How much of recognition errors is caused by search errors?
• We can estimate this from a sample test data, for which the correct answer is known, as follows:
  – For each utterance \( j \) in the test set, run recognition using pruning and note the best cost \( C_j' \) obtained for the result
  – For each utterance \( j \), also match the correct word to the input without pruning, and note its cost \( C_j \)
  – If \( C_j \) is better than \( C_j' \) we have a pruning error or search error for utterance \( j \)
• Pruning errors can be reduced by lowering the pruning threshold (\( i.e. \) making it less aggressive)
• Note, however, this does not guarantee that the correct word is recognized!
  – The new pruning threshold may uncover other incorrect paths that perform better than the correct one
Summary So Far

- Dynamic programming for finding minimum cost paths
- Trellis as realization of DP, capturing the search dynamics
  - Essential components of trellis
- DP applied to string matching
- Adaptation of DP to template matching of speech
  - Dynamic Time Warping, to deal with varying rates of speech
- Isolated word speech recognition based on template matching
- Time synchronous search
- Isolated word recognition using automatic endpointing
- Dealing with errors using confidence estimation and N-best lists
- Improving recognition accuracy through multiple templates
- Beam search and beam pruning
A Footnote: Reversing Sense of “Cost”

- So far, we have a *cost* measure in DP and DTW, where higher values imply worse match.
- We will also frequently use the opposite kind, where higher values imply a *better* match; *e.g.*:
  - The same cost function but with the sign changed (*i.e. negative* Euclidean distance\(= -\sqrt{\sum (x_i - y_i)^2}\); *X* and *Y* being vectors)
  - \(- \sum (x_i - y_i)^2\); *i.e.* – Euclidean distance squared

- We may often use the generic term *score* to refer to such values
  - Higher scores imply better match
DTW Using Scores

• How should DTW be changed when using scores vs costs?
• At least three points to consider:
  – Obviously, we need to maximize the total path score, rather than minimize it
  – Beam search must be adjusted as follows: if the best partial path score achieved in a frame is $X$, prune away all nodes with partial path score $< X–T$
    • instead of $> X+T$
    • where $T$ is the beam pruning threshold)
  – Likewise, in confidence estimation, we accept paths with scores above the confidence threshold
  – in contrast to cost values below the threshold
Likelihood Functions for Scores

• Another common method is to use a probabilistic function, for the local node or edge “costs” in the trellis
  – Edges have transition probabilities
  – Nodes have output or observation probabilities
    • They provide the probability of the observed input
  – Again, the goal is to find the template with highest probability of matching the input

• Probability values as “scores” are also called likelihoods
Gaussian Distribution as Likelihood Function

- If $x$ is an input feature vector and $\mu$ is a template vector of dimensionality $N$, the function:

$$f_X(x_1, \ldots, x_n) = \frac{1}{(2\pi)^{N/2} |\Sigma|^{1/2}} \exp \left( -\frac{1}{2} (x - \mu) \Sigma^{-1} (x - \mu) \right)$$

is the famous multivariate Gaussian distribution, where $\Sigma$ is the co-variance matrix of the distribution.

- It is one of the most commonly used probability distribution functions for acoustic models in speech recognition.

- We will look at this in more detail later.
DTW Using Probabilistic Values

- As with scores (negative-cost) we must maximize the total path likelihood, since higher likelihoods $\Rightarrow$ better match

- However, the total likelihood for a path is the \textit{product} of the local node and edge likelihoods, rather than the sum
  - One multiplies the individual probabilities to obtain a joint probability value

- As a result, beam pruning has to be modified as follows:
  - if the best partial path likelihood in a frame is $X$, prune all nodes with partial path likelihood $< XT$
    - $T$ is the beam pruning threshold
  - Obviously, $T < 1$
Log Likelihoods

- Sometimes, it is easier to use the logarithm of the likelihood function for scores, rather than likelihood function itself.
- Such scores are usually called log-likelihood values.
  - Using log-likelihoods, multiplication of likelihoods turns into addition of log-likelihoods, and exponentiation is eliminated.
- Many speech recognizers operate in log-likelihood mode.
Some Fun Exercises with Likelihoods

• How should the DTW algorithm be modified if we use log-likelihood values instead of likelihoods?

• Application of technique known as scaling:
  – When using cost or score (-ve cost) functions, show that adding some arbitrary constant value to all the partial path scores in any given frame does not change the outcome
    • The constant can be different for different input frames
  – When using likelihoods, show that multiplying partial path values by some positive constant does not change the outcome

• If the likelihood function is the multivariate Gaussian with identity covariance matrix (i.e. the $\Sigma$ term disappears), show that using the log-likelihood function is equivalent to using the Euclidean distance squared cost function